correction (Interrogation : complexes)

Exercice 1

effectuez les opérations suivantes en écriture algébrique:

a) $z_1 + z_2 = 2 + \left(\sqrt{3} - 5\right)i$

b) $z_1 \times z_2 = \left(-15 + 5\sqrt{3}\right) + \mathrm{i}\left(5\sqrt{3} + 15\right)$

c) $z_1 \div z_2 = \frac{(5-5\mathrm{i})}{(-3+\sqrt{3}\mathrm{i})} = \frac{(5-5\mathrm{i})(-3-\sqrt{3}\mathrm{i})}{(-3+\sqrt{3}\mathrm{i})(-3-\sqrt{3}\mathrm{i})} = \frac{\left(-15-5\sqrt{3}\right)+\mathrm{i}\left(-5\sqrt{3}+15\right)}{12} = \frac{-15-5\sqrt{3}}{12} + i\frac{-5\sqrt{3}+15}{12}$

Exercice 2

donnez l'écriture trigonométrique de z1 et z2

$|z_1| = 5\sqrt{2}$ $\cos(\theta_1) = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ et $\sin(\theta_1) = \frac{-5}{5\sqrt{2}} = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$ $\arg(z_1) = \theta_1 = \frac{-\pi}{4}$

$|z_2| = 2\sqrt{3}$ $\cos(\theta_2) = \frac{-3}{2\sqrt{3}} = \frac{-3\sqrt{3}}{2\times 3} = \frac{\sqrt{3}}{2}$ et $\sin(\theta_2) = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}$ $\arg(z_2) = \theta_2 = \frac{5\pi}{6}$

Ainsi $z_1 = \left[5\sqrt{2}; \frac{-\pi}{4}\right]$ et $z_2 = \left[2\sqrt{3}; \frac{5\pi}{6}\right]$

donnez les écritures trigonométriques de :

a) $z_1 \times z_2 = \left[5\sqrt{2}; \frac{-\pi}{4}\right]\left[2\sqrt{3}; \frac{5\pi}{6}\right] = \left[5\sqrt{2}\times 2\sqrt{3}; \frac{-\pi}{4} + \frac{5\pi}{6}\right] = \left[10\sqrt{6}; \frac{7\pi}{12}\right]$

b) $z_1 \div z_2 = \frac{\left[5\sqrt{2}; \frac{-\pi}{4}\right]}{\left[2\sqrt{3}; \frac{5\pi}{6}\right]} = \left[\frac{5\sqrt{2}}{2\sqrt{3}}; \frac{-\pi}{4} - \frac{5\pi}{6}\right] = \left[\frac{5}{2}\sqrt{\frac{2}{3}}; \frac{-13\pi}{12}\right] = \left[\frac{5}{2}\frac{\sqrt{2}}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}; \frac{-13\pi}{12}\right] = \left[\frac{5\sqrt{6}}{6}; \frac{-13\pi}{12}\right]$

c) $\frac{1}{z_1} = \frac{1}{\left[5\sqrt{2}; \frac{-\pi}{4}\right]} = \frac{1}{\left[5\sqrt{2}; \frac{\pi}{4}\right]} = \left[\frac{1}{5\sqrt{2}}; \frac{-\pi}{4}\right] = \left[\frac{\sqrt{2}}{10}; \frac{-\pi}{4}\right]$

d) $z_2{}^6 = \left[\left(2\sqrt{3}\right)^6; \frac{5\pi}{6}6\right]$ = [64×27;5 π] = [1728; π]

Exercice 3

Résoudre l'équation suivante : (z+3i – 5)(2z² - 6z +7)=0

Je résous 2z² - 6z +7=0

$\Delta = (-6)^2 - 4\times 2\times 7 = 36 - 56 = 20$

$\Delta < 0$ nous aurons donc deux solutions complexes conjuguées :

$z_1 = \frac{6-i\sqrt{20}}{4} = \frac{6-i2\sqrt{5}}{4} = \frac{3-i\sqrt{5}}{2}$ et donc $z_2 = \frac{3+i\sqrt{5}}{2}$

Je résous z + 3i -5 = 0 on aura z = 5 -3i

Ainsi l'équation (z+3i – 5) (2z² - 6z +7)=0 a pour solutions : 5 -3i, $\frac{3-i\sqrt{5}}{2}$ et $\frac{3+i\sqrt{5}}{2}$

Exercice 4

a) $\cos\left(\frac{7\pi}{12}\right) = \frac{-15+5\sqrt{3}}{10\sqrt{6}} = \frac{-3+\sqrt{3}}{2\sqrt{6}} = \frac{-3\sqrt{6}+\sqrt{3}\sqrt{6}}{2\sqrt{6}\sqrt{6}} = \frac{-3\sqrt{6}+3\sqrt{2}}{12} = \frac{-\sqrt{6}+\sqrt{2}}{4}$

b) $\sin\left(\frac{7\pi}{12}\right) = \frac{15+5\sqrt{3}}{10\sqrt{6}} = \frac{3+\sqrt{3}}{2\sqrt{6}} = \frac{3\sqrt{6}+\sqrt{3}\sqrt{6}}{2\sqrt{6}\sqrt{6}} = \frac{3\sqrt{6}+3\sqrt{2}}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$

c) $\cos\left(\frac{-13\pi}{12}\right) = \frac{\frac{\left(-15-5\sqrt{3}\right)}{12}}{\frac{5\sqrt{6}}{6}} = \frac{\frac{\left(-15-5\sqrt{3}\right)}{12}12\sqrt{6}}{\frac{5\sqrt{6}}{6}12\sqrt{6}} = \frac{-15\sqrt{6} - 5\sqrt{6}\sqrt{3}}{60} = \frac{-15\sqrt{6} - 15\sqrt{2}}{60} = \frac{-\sqrt{6} - \sqrt{2}}{4}$

d) $\sin\left(\frac{-13\pi}{12}\right) = \frac{\frac{\left(15-5\sqrt{3}\right)}{12}}{\frac{5\sqrt{6}}{6}} = \frac{\frac{\left(15-5\sqrt{3}\right)}{12}12\sqrt{6}}{\frac{5\sqrt{6}}{6}12\sqrt{6}} = \frac{15\sqrt{6} - 5\sqrt{6}\sqrt{3}}{60} = \frac{15\sqrt{6} - 15\sqrt{2}}{60} = \frac{\sqrt{6} - \sqrt{2}}{4}$