Correction d'exercices d'entrainement à la recherche de Primitives

Exercice 55P200

1) $F(x)=\frac{\left(x^2+3\right)^5}{5}$ 2) $F(x)=\frac{\left(x^3-2x+3\right)^4}{4}$ 3) $F(x)=\frac{\left(x^2+x+1\right)^6}{6}$

Exercice 56P200

1) $f(x)=-1(-\sin x)(\cos x)^4$ 2) $f(x)=\frac{1}{2}(2x+2)(x^2+2x-5)^4$ $f(x)=-\frac{1}{2}(2x-2)(x^2+2x-5)^5$

$F(x)=-\frac{(\cos x)^5}{5}$ $F(x)=\frac{1}{2}\frac{\left(x^2+2x-5\right)^5}{5}$ $F(x)=-\frac{1}{2}\frac{\left(x^2+2x-5\right)^6}{6}$

Exercice 57P200

1) $f(x)=\frac{1}{2}(6x-2)(3x^2-2x+3)^3$ $F(x)=\frac{1}{2}\frac{\left(3x^2-2x+3\right)^4}{4}$

2) $f(x)=\frac{2}{3}(3x^2-3)(x^3-3x+4)^5$ $F(x)=\frac{2}{3}\frac{\left(x^3-3x+4\right)^6}{6}$

3)$F(x)=\frac{(4x+3)^4}{4}$

4) $f(x)=-\frac{1}{2}(-2)(-2x+1)^4$ $F(x)=-\frac{1}{2}\frac{(-2x+1)^5}{5}$

Exercice 59P200

1) $f(x)=\tan x=\frac{\sin x}{\cos x}$ donc $f'(x)=\frac{\cos x\cos x-(-\sin x)\sin x}{(\cos x)^2}=\frac{(\cos x)^2}{(\cos x)^2}+\frac{(\sin x)^2}{(\cos x)^2}=1+\left(\frac{\sin x}{\cos x}\right)^2=1+(\tan x)^2$

2) $g(x)=(1+(\tan x)^2)(\tan x)^4$ on a donc ici $u'u^4$ et donc $G(x)=\frac{(\tan x)^5}{5}$

3) erreur d'énoncé : il fallait lire $f'(x)=\frac{1}{\cos^2 x}$,

$f'(x)=\frac{\cos x\cos x-(-\sin x)\sin x}{(\cos x)^2}=\frac{(\cos x)^2+(\sin x)^2}{(\cos x)^2}=\frac{1}{(\cos x)^2}$

4) $h(x)=\frac{1}{(\cos x)^2}(\tan x)^3$ donc $H(x)=\frac{(\tan x)^4}{4}$

Exercice 60P200

1) $f(x)=\tan^4 x\ \ (1+\tan^2 x)$ on a donc d'après la question 1 du 59 la forme : u^4u' ainsi $F(x)=\frac{(\tan x)^5}{5}$

2) on sait que $(f\circ g)'(x)=g'(x)\ (f'\circ g)(x)$ donc la dérivée de $\tan 3x$ est : $3(1+\tan^2 3x)$

Ainsi $f(x)=\tan 3x+\tan^3 3x=\tan 3x\,(1+\tan^2 3x)=\frac{1}{3}\big(3(1+\tan^2 3x)\big)\tan 3x$ j'ai fait apparaitre du $u'u^1$

Donc $F(x)=\frac{1}{3}\frac{(\tan 3x)^2}{2}$

Exercice 61P200

1) $\sin^3 x=\sin x\sin^2 x=\sin x\ \ (1-\cos^2 x)=\sin x-\sin x\ \ \cos^2 x$

2) $f(x)=\sin^3 x=\sin x-\sin x\ \ \cos^2 x=\sin x+(-\sin x)\ \ \cos^2 x$

Donc $F(x)=-\cos x+\frac{\cos^3 x}{3}$

Exercice 62P200

1) $F(x)=-\frac{1}{3(x^2+5x)^3}$ 2) $F(x)=-\frac{1}{x^3+2x}$ 3) $f(x)=\frac{1}{2}\frac{2x}{(x^2+1)^3}$ $F(x)=\frac{1}{2}\frac{-1}{2(x^2+1)^2}$

Exercice 63P200

1) $F(x) = -\frac{7}{3(x-1)^3}$

2) $f(x) = \frac{9}{4}\frac{4}{(4x+1)^3}$ $\quad F(x) = \frac{9}{4}\ \frac{-1}{2(4x+1)^2} = \frac{-9}{8(4x+1)^2}$

3) $f(x) = \frac{1}{3}\frac{2}{(2x+3)^2}$ $\quad F(x) = \frac{1}{3}\ \frac{-1}{2x+3} = -\frac{1}{3(2x+3)}$

Exercice 64P200

1) $f(x) = \frac{1-x^2}{(x^3-3x+1)^3} = -\frac{1}{3}\ \frac{3x^2-3}{(x^3-3x+1)^3}$ donc $F(x) = -\frac{1}{3}\ \frac{-1}{2(x^3-3x+1)^2} = \frac{1}{6(x^3-3x+1)^2}$

2) $F(x) = -\frac{1}{3(x^3+2x)^3}$

3) $f(x) = -\frac{1}{12}\frac{2x-4}{(x^2-4x+9)^4}$ donc $F(x) = -\frac{1}{12}\frac{-1}{3(x^2-4x+9)^3} = \frac{1}{36(x^2-4x+9)^3}$

Exercice 65P201

1) $f(x) = \frac{1}{6}\frac{6x-6}{(3x^2-6x+11)^7}$ $\quad F(x) = \frac{1}{6}\ \frac{-1}{6(3x^2-6x+11)^6} = \frac{-1}{36(3x^2-6x+11)^6}$

2) $f(x) = -2\frac{2x-3}{(x^2-3x-4)^5}$ $\quad F(x) = -2\ \frac{-1}{4(x^2-3x-4)^4} = \frac{1}{2(x^2-3x-4)^4}$

Exercice 66P201

1) $f(x) = \frac{-1}{3}\frac{3x^2-3}{(x^3-3x+1)^3}$ $\quad F(x) = \frac{-1}{3}\ \frac{-1}{2(x^3-3x+1)^2} = \frac{1}{6(x^3-3x+1)^2}$

2) $f(x) = -5\frac{3x^2+2}{(x^3+2x)^4}$ $\quad F(x) = -5\frac{-1}{3(x^3+2x)^3} = \frac{5}{3(x^3+2x)^3}$

3) $f(x) = \frac{\cos x}{\sin^3 x}$ ici on a : $u = \sin x$, $u' = \cos x$ et vu que $\frac{u'}{u^3} \to -\frac{1}{2u^2}$ on aura $F(x) = \frac{-1}{2\sin^2 x}$

Exercice 67P201

1) $f(x) = \frac{\tan x}{(1+(\tan x)^2)^2} = \frac{\tan x}{\left(\frac{1}{\cos^2 x}\right)^2} = \tan x \cos^4 x = \sin x \cos^3 x = -1\,(-\sin x)\cos^3 x$

Vu que $u'u^3 \to \frac{u^4}{4}$ on aura donc $F(x) = -1\frac{\cos^4 x}{4}$

2) $f(x) = -1\frac{-\sin x}{\cos^3 x}$ $\quad$ donc $F(x) = -1\frac{-1}{2\cos^2 x} = \frac{1}{2\cos^2 x}$

3) $f(x) = \frac{\cos x}{\sin^3 x(1-\cos^2 x)^6} = \frac{\cos x}{\sin^3 x(\sin^2 x)^6} = \frac{\cos x}{\sin^{15} x}$ $\quad$ donc $F(x) = \frac{-1}{14\sin^{14} x}$

Exercice 68P201

1) $F(x) = 2\sqrt{3x+1}$

2) $f(x) = -\frac{1}{5}\frac{(-5)}{\sqrt{4-5x}}$ $\quad$ donc $F(x) = -\frac{1}{5}2\sqrt{4-5x}$

3) $f(x) = \frac{1}{2}\frac{2x}{\sqrt{x^2+1}}$ $\quad$ donc $F(x) = \frac{1}{2}2\sqrt{x^2+1}$

Exercice 69P201

1) $f(x) = \frac{5}{2}\ \frac{2x}{\sqrt{2x+1}}$ $\quad$ donc $F(x) = \frac{5}{2}\ 2\sqrt{2x+1} = 5\sqrt{2x+1}$

2) $f(x) = -2\ \frac{1}{\sqrt{x-6}}$ $\quad$ donc $F(x) = -2 \times 2\sqrt{x-6} = -4\sqrt{x-6}$

3) $f(x) = \frac{x-1}{\sqrt{x(x-2)}} = \frac{x-1}{\sqrt{x^2-2x}} = \frac{1}{2}\frac{2x-2}{\sqrt{x^2-2x}}$ donc $F(x) = \frac{1}{2}2\sqrt{x^2-2x} = \sqrt{x^2-2x}$

Exercice 70P201

1) $f(x) = \frac{x+1}{\sqrt{x^2+2x+5}} = \frac{1}{2}\ \frac{2x+2}{\sqrt{x^2+2x+5}}$ $\quad$ donc $F(x) = \frac{1}{2}\ 2\sqrt{x^2+2x+5} = \sqrt{x^2+2x+5}$

2) $f(x) = \frac{5}{6}\frac{6x^2+6x+6}{\sqrt{2x^3+3x^2+6x+1}}$ $\quad$ donc $F(x) = \frac{5}{6}2\sqrt{2x^3+3x^2+6x+1} = \frac{5}{3}\sqrt{2x^3+3x^2+6x+1}$

Exercice 71P201

1) $f(x) = \frac{1}{3}\ \frac{6x^3+3\sin x}{\sqrt{2x^3-3\cos x+3}}$ donc $F(x) = \frac{1}{3}2\sqrt{2x^3 - 3\cos x + 3}$

2) $\sin(2x) = 2\sin x\cos x$ donc $f(x) = \frac{2\sin x\cos x}{\sqrt{1+\sin^2 x}}$ donc $F(x) = 2\sqrt{1+\sin^2 x}$

Exercice 77P201

1) $f(x) = 3x - 1$ donc $F(x) = \frac{3x^2}{2} - x + c$ de plus on veut $F(-1) = 3 \Leftrightarrow \frac{3(-1)^2}{2} + 1 + c = 3$

$\Leftrightarrow c = \frac{1}{2}$ donc $F(x) = \frac{3x^2}{2} - x + \frac{1}{2}$

2) $f(x) = x - \frac{1}{x^2}$ donc $F(x) = \frac{x^2}{2} + \frac{1}{x} + c$ de plus on veut $F(1) = -1$ $\Leftrightarrow \frac{1^2}{2} + \frac{1}{1} + c = -1$

$\Leftrightarrow c = -\frac{5}{2}$ donc $F(x) = \frac{x^2}{2} + \frac{1}{x} - \frac{5}{2}$

Exercice 80P202

2) $f(x) = \frac{1}{2}\frac{2}{(2x+5)^4}$ donc $F(x) = \frac{1}{2}\ \frac{-1}{3(2x+5)^3} + c = \frac{-1}{6(2x+5)^3} + c$ de plus $F(-3) = 0$

$\Leftrightarrow \frac{-1}{6(-1)^3} + c = 0 \Leftrightarrow c = -\frac{1}{6}$ donc $F(x) = \frac{-1}{6(2x+5)^3} - \frac{1}{6}$

3) $f(x) = x^2 + 3 - \frac{2}{x^2}$ donc $F(x) = \frac{x^3}{3} + 3x + \frac{2}{x} + c$ de plus $F(2) = -1$

$\Leftrightarrow \frac{8}{3} + 6 + \frac{2}{2} + c = -1 \Leftrightarrow c = -\frac{32}{3}$ donc$F(x) = \frac{x^3}{3} + 3x + \frac{2}{x} - \frac{32}{3}$

4) $f(x) = -1(-\sin x)\cos^3 x$ donc $F(x) = -1\frac{\cos^4 x}{4} + c$ de plus $F(\pi) = 0$

$\Leftrightarrow 0 = -1\frac{\cos^4 \pi}{4} + c \Leftrightarrow 0 = -\frac{1}{4} + c \Leftrightarrow c = \frac{1}{4}$

Exercice 86P202

1) $\frac{a}{(x+3)^2} + \frac{b}{(x+3)^3} = \frac{a(x+3)+b}{(x+3)^3} = \frac{ax+3a+b}{(x+3)^3}$

Sur D_f on a : $f(x) = \frac{a}{(x+3)^2} + \frac{b}{(x+3)^3} \Leftrightarrow \frac{2x+3}{(x+3)^3} = \frac{ax+3a+b}{(x+3)^3} \Leftrightarrow 2x+3 = ax + 3a + b \Leftrightarrow \begin{cases} 2 = a \\ 3 = 3a + b \end{cases} \Leftrightarrow \begin{cases} 2 = a \\ -6 = b \end{cases}$

Ainsi on $f(x) = \frac{2}{(x+3)^2} + \frac{-6}{(x+3)^3}$

2) sur $]-\infty;-3[$ on aura $F(x) = \frac{-2}{x+3} + \frac{6}{2(x+3)^2} + c$

Comme on veut : $F(-4) = 0$ on aura $0 = \frac{-2}{-1} + \frac{6}{2(-1)^2} + c \Leftrightarrow c = -5$

Ainsi : $F(x) = \frac{-2}{x+3} + \frac{6}{2(x+3)^2} - 5$