# Dérivations : Fiche d'exercice n°1 (correction)

**Exercice 1**

a) $D_f = \mathbb{R}, f(x) = x + 1$
$f'(x) = 1$

b) $D_f = \mathbb{R}, f(x) = \frac{2}{3} - x$
$f'(x) = -1$

c) $D_f = \mathbb{R}, f(x) = \frac{x}{2}$
$f'(x) = \frac{1}{2}$

d) $D_f = ]0; +\infty[, f(x) = \frac{\sqrt{x}}{2}$
$f'(x) = \frac{1}{2}\frac{1}{2\sqrt{x}} = \frac{1}{4\sqrt{x}}$

e) $D_f = \mathbb{R}^*, f(x) = \frac{2}{x} + x^2 + 1$
$f'(x) = \frac{-2}{x^2} + 2x$

f) $D_f = \mathbb{R}, f(x) = -\frac{3}{2x} = -\frac{3}{2}\frac{1}{x}$
$f'(x) = \frac{-3}{2x^2}$

g) $D_f = ]0; +\infty[, f(x) = x^5 + \frac{1}{x} + \sqrt{x}$
$f'(x) = 5x^4 + \frac{-1}{x^2} + \frac{1}{2\sqrt{x}}$

h) $D_f = \mathbb{R}, f(x) = \frac{2x^2+1}{x}$
$f'(x) = \frac{(4x)x-(2x^2+1)1}{x^2} = \frac{2x^2-1}{x^2}$

i) $D_f = \mathbb{R}, f(x) = 3x^4 - \frac{2}{3}x^3$
$f'(x) = 12x^3 - 2x^2$

j)$D_f = \mathbb{R}^*, f(x) = -\frac{3}{7x} + \frac{3}{2}x^2$
$f'(x) = \frac{3}{7x^2} + 3x$

k) $D_f = ]0; +\infty[, f(x) = x\sqrt{x}$
$f'(x) = 1\sqrt{x} + x\frac{1}{2\sqrt{x}} = \frac{3}{2}\sqrt{x}$

l) $D_f = \mathbb{R}^*, f(x) = \frac{x^{-4}}{8}$
$f'(x) = -4\frac{x^{-5}}{8} = \frac{x^{-5}}{-2} = -\frac{1}{2x^5}$

m) $D_f = \mathbb{R}\setminus\left\{\frac{2}{3}\right\}, f(x) = \frac{3x-1}{2-3x}$
$f'(x) = \frac{3(2-3x)-(3x-1)(-3)}{(2-3x)^2} = \frac{6-9x+9x-3}{(2-3x)^2} = \frac{3}{(2-3x)^2}$

**Exercice 2**

Comparer pour chaque expression la dérivée de la version factorisée et celle de la forme développée.

a) $f(x) = (3x-1)(x-1)$ $\quad f'(x) = 3(x-1) + (3x-1)1 = 6x - 4$
en développant : $f(x) = 3x^2 - 4x + 1$ $\quad f'(x) = 6x - 4$
b) $f(x) = (1-3x)(x^2-x)$ $\quad f'(x) = -3(x^2-x) + (1-3x)(2x-1)$
$= -3x^2 + 3x - 1 + 5x - 6x^2 = -9x^2 + 8x - 1$
En développant : $f(x) = x^2 - x - 3x^3 + 3x^2 = -3x^3 + 4x^2 - x$
On dérive : $f'(x) = -9x^2 + 8x - 1$
c) $f(x) = \left(\frac{1}{x} - \frac{x}{2}\right)(2x^2+1)$ $\quad f'(x) = \left(-\frac{1}{x^2} - \frac{1}{2}\right)(2x^2+1) + \left(\frac{1}{x} - \frac{x}{2}\right)(4x)$
$= \left(-2 - \frac{1}{x^2} - x^2 - \frac{1}{2}\right) + (4 - 2x^2) = -3x^2 + \frac{3}{2} - \frac{1}{x^2}$
En développant : $f(x) = 2x + \frac{1}{x} - x^3 - \frac{x}{2} = -x^3 + \frac{3}{2}x + \frac{1}{x}$ donc on a bien $f'(x) = -3x^2 + \frac{3}{2} - \frac{1}{x^2}$

**Exercice 3**

Pour chaque fonction f , calculez la dérivée de f puis celle de $\frac{1}{f}$ : a) $f(x) = 3x^2 - 1$, b) $f(x) = \sqrt{x}$ , c) $f(x) = \frac{3x-2}{x}$

a) $f'(x) = 6x$,
$\frac{1}{f(x)} = \frac{1}{3x^2-1}$
$\left(\frac{1}{f}\right)'(x) = \frac{-6x}{(3x^2-1)^2}$

b) $f'(x) = \frac{1}{2\sqrt{x}}$ ,
$\frac{1}{f(x)} = \frac{1}{\sqrt{x}}$
$\left(\frac{1}{f}\right)'(x) = \frac{\left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x})^2} = \frac{1}{2x\sqrt{x}}$

c) $f'(x) = \frac{3x-(3x-2)1}{x^2} = \frac{2}{x^2}$
$\frac{1}{f(x)} = \frac{x}{3x-2}$
$\left(\frac{1}{f}\right)'(x) = \frac{1(3x-2)-3x}{(3x-2)^2} = \frac{-2}{(3x-2)^2}$

**Exercice 4**

Pour chaque fonction dériver, puis déterminer l'ensemble de définition de la fonction et de sa dérivée.

a) $f(x) = x^3 - \frac{1}{x}$
$D_f = \mathbb{R}^*$
$f'(x) = 3x^2 + \frac{1}{x^2}$
$D_{f'} = \mathbb{R}^*$

b) $f(x) = 2x + \sqrt{x}$
$D_f = [0; +\infty[$
$f'(x) = 2 + \frac{1}{2\sqrt{x}}$
$D_{f'} = ]0; +\infty[$

c) $f(x) = 3x^2 - \frac{x}{2} + \frac{4}{5}$
$D_f = \mathbb{R}$
$f'(x) = 6x - \frac{1}{2}$
$D_{f'} = \mathbb{R}$

d) $f(x) = \frac{x}{3} - \frac{3}{x}$
$D_f = \mathbb{R}^*$
$f'(x) = \frac{1}{3} - \frac{3}{x^2}$
$D_{f'} = \mathbb{R}^*$

e) $f(x) = \frac{2}{2x}$
$D_f = \mathbb{R}^*$

f) $f(x) = \sqrt{\frac{4}{9}}\sqrt{x} = \frac{2}{3}\sqrt{x}$
$D_f = [0; +\infty[$

g) $f(x) = (x^2+1)\sqrt{x}$
$D_f = [0; +\infty[$

h) $f(x) = \frac{x^3}{3} - \frac{x^4}{2}$
$D_f = \mathbb{R}$

$f'(x) = \frac{-1}{x^2}$
$D_{f'} = \mathbb{R}^*$

$f'(x) = \frac{2}{3}\frac{1}{2\sqrt{x}} = \frac{1}{3\sqrt{x}}$
$D_{f'} = [0; +\infty[$

$f'(x) = 2x\sqrt{x} + \frac{x^2+1}{2\sqrt{x}}$
$D_{f'} = ]0; +\infty[$

$f'(x) = 3x^2 - 2x^3$
$D_{f'} = \mathbb{R}$

i) $f(x) = (x^2+1)(1+\sqrt{x})$ $D_f = [0; +\infty[$ $f'(x) = 2x(1+\sqrt{x}) + (x^2+1)\frac{1}{2\sqrt{x}} = 2x + 2x\sqrt{x} + \frac{1}{2}x\sqrt{x} + \frac{1}{2\sqrt{x}}$
$f'(x) = 2x + \frac{5}{2}x\sqrt{x} + \frac{1}{2\sqrt{x}}$ donc $D_{f'} = ]0; +\infty[$

j) $f(x) = \frac{3x-2}{x+4}$
$D_f = \mathbb{R}\backslash\{-4\}$
$f'(x) = \frac{3(x+4)-(3x-2)1}{(x+4)^2} = \frac{14}{(x+4)^2}$
$D_{f'} = \mathbb{R}\backslash\{-4\}$

k) $f(x) = \frac{-2x+1}{x-3}$
$D_f = \mathbb{R}\backslash\{3\}$
$f'(x) = \frac{-2(x-3)-(-2x+1)1}{(x-3)^2} = \frac{5}{(x-3)^2}$
$D_{f'} = \mathbb{R}\backslash\{3\}$

l) $f(x) = \frac{x^2-1}{x^2+2}$
$D_f = \mathbb{R}$
$f'(x) = \frac{2x(x^2+2)-(x^2-1)2x}{(x^2+2)^2} = \frac{6x}{(x^2+2)^2}$
$D_{f'} = \mathbb{R}$

m) $f(x) = \frac{1}{1-x}$
$D_f = \mathbb{R}\backslash\{1\}$
$f'(x) = \frac{1}{(1-x)^2}$
$D_{f'} = \mathbb{R}\backslash\{1\}$

n) $f(x) = \frac{5x^2-3x+2}{2x^2-x-1}$ $\Delta = (-1)^2 - 4 \times 2 \times (-1) = 9$
$x_1 = \frac{1-\sqrt{9}}{4} = -\frac{1}{2}$ et $x_2 = \frac{1+\sqrt{9}}{4} = 1$ donc $D_f = \mathbb{R}\backslash\left\{-\frac{1}{2}; 1\right\}$
$f'(x) = \frac{(10x-3)(2x^2-x-1)-(5x^2-3x+2)(4x-1)}{(2x^2-x-1)^2}$
$= \frac{10x^3-10x^2-10x-6x^2+3x+3-(20x^3-5x^2-12x^2+3x+8x-2)}{(2x^2-x-1)^2} = \frac{-10x^3+x^2-18x+5}{(2x^2-x-1)^2}$
$D_{f'} = \mathbb{R}\backslash\left\{-\frac{1}{2}; 1\right\}$