

Exercice 1 : Dérivées des fonctions

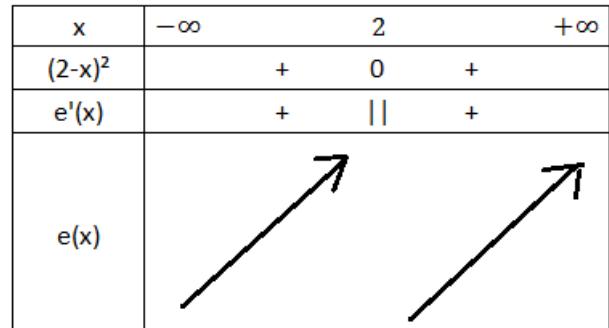
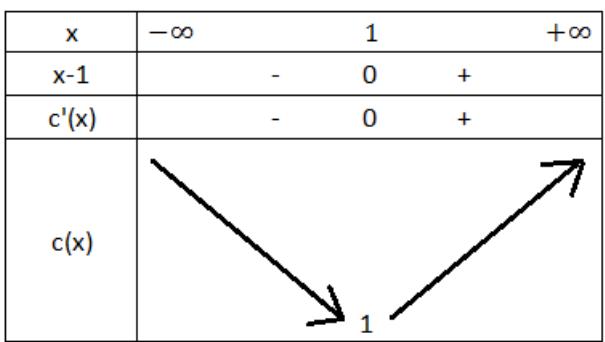
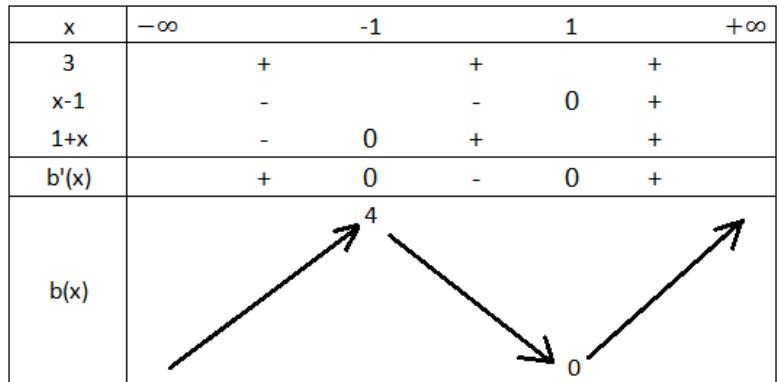
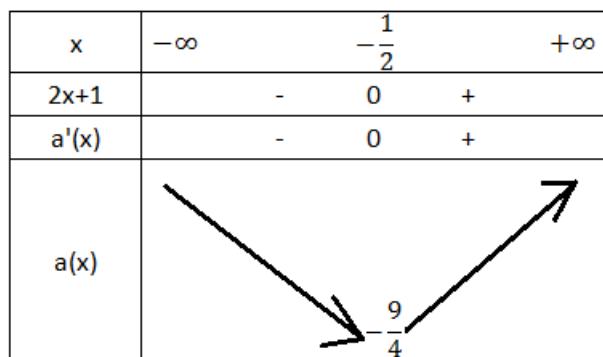
$$\begin{array}{lll}
 a(x) = 3x + 4 & a'(x) = 3 & b(x) = x^7 - x^3 \quad b'(x) = 7x^6 - 3x^2 \\
 c(x) = 4x^3 + 2x^2 + 5x - 7 & c'(x) = 12x^2 + 4x + 5 & d(x) = 17\sqrt{x} - 51x \quad d'(x) = \frac{17}{2\sqrt{x}} - 51 \\
 e(x) = \frac{17}{x} + 5 - 2x & e'(x) = -\frac{17}{x^2} - 2 & f(t) = t^2\sqrt{t} \quad f'(t) = 2t\sqrt{t} + \frac{t^2}{2\sqrt{t}} \\
 g(t) = (3t+7)(5-4t)(3t) & g'(t) = [3(5-4t) + (3t+7)(-4)](3t) + (3t+7)(5-4t)3 \\
 = [15-12t-12t-28]3t + (15t-12t^2+35-28t)3 = -39t-72t^2-39t-36t^2+105 = -108t^2-78t+105
 \end{array}$$

Version alternative :

$$\begin{aligned}
 g(t) &= (3t+7)(5-4t)(3t) = (15t-12t^2+35-28t)3t = -36t^3-39t^2+105 \text{ donc } g'(t) = -108t^2-78t+105 \\
 h(l) &= (l^2-1)(l^2+1) = l^4-1 \text{ donc } h'(l) = 4l^3 \\
 i(x) &= \left(\frac{1}{x}+4\right)(1-x) = \frac{1}{x}-1+4-4x \quad i'(x) = -4-\frac{1}{x^2} \quad j(x) = \frac{1}{3x+4} \quad j'(x) = -\frac{3}{(3x+4)^2} \\
 k(\mu) &= 4x-\frac{7}{\mu^2+1} \quad k'(\mu) = \frac{14\mu}{(\mu^2+1)^2} \text{ attention } x \text{ ici est une constante et non une variable idem pour } p(t) \\
 l(x) &= \frac{1}{x}+\frac{1}{x+1}+\frac{1}{x+2} \quad l'(x) = \frac{-1}{x^2}+\frac{-1}{(x+1)^2}+\frac{-1}{(x+2)^2} \quad m(x) = \frac{4x-6}{3x+4} \quad m'(x) = \frac{4(3x+4)-(4x-6)3}{(3x+4)^2} = \frac{12x+16-12x+18}{(3x+4)^2} = \frac{34}{(3x+4)^2} \\
 n(x) &= \frac{x-\sqrt{3}}{x+\sqrt{3}} \quad n'(x) = \frac{(x+\sqrt{3})-(x-\sqrt{3})}{(x+\sqrt{3})^2} = \frac{2\sqrt{3}}{(x+\sqrt{3})^2} \quad p(t) = \frac{x^2-3x+2}{x^2+x+4} \quad p'(t) = 0 \\
 o(n) &= n^3 - \frac{n+4}{n^2+3n-4} \quad o'(n) = 3n^2 - \frac{(n^2+3n-4)-(n+4)(2n+3)}{(n^2+3n-4)^2} = 3n^2 - \frac{-n^2-8n-16}{(n^2+3n-4)^2} \quad q(r) = r^{24} \quad q'(r) = 24r^{23} \\
 r(q) &= q - \frac{1}{q+1} \quad r'(q) = 1 - \frac{-1}{(q+1)^2} \\
 s(o) &= \frac{o^3-o^2+o}{o+1} \quad s'(o) = \frac{(3o^2-2o+1)(o+1)-(o^3-o^2+o)1}{(o+1)^2} = \frac{3o^3+o^2-o+1-o^3+o^2-o}{(o+1)^2} = \frac{2o^3+2o^2-2o+1}{(o+1)^2}
 \end{aligned}$$

Exercice 2 : Fais les tableaux de variation des fonctions suivantes sur leurs domaines de définition.

$$\begin{array}{lll}
 a(x) = x^2 + x - 2 & a'(x) = 2x + 1 & \text{changement de signe en } -\frac{1}{2} \\
 b(x) = x^3 - 3x + 2 & b'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x-1)(x+1) \\
 c(x) = 3x^2 - 6x + 4 & c'(x) = 6x - 6 = 6(x-1) \\
 d(x) = -2x^3 + \frac{7}{2}x^2 - 2x + 1 & d'(x) = -6x^2 + 7x - 2 \quad \Delta = 1 \quad x_1 = \frac{-7-\sqrt{1}}{-12} = \frac{-8}{-12} = \frac{2}{3} \quad x_2 = \frac{-7+\sqrt{1}}{-12} = \frac{-6}{-12} = \frac{1}{2} \\
 & \text{Donc } d'(x) = -6\left(x-\frac{1}{2}\right)\left(x-\frac{2}{3}\right) \\
 e(x) = \frac{x-1}{2-x} & e'(x) = \frac{1(2-x)-(x-1)(-1)}{(2-x)^2} = \frac{2-x+x-1}{(2-x)^2} = \frac{1}{(2-x)^2} \\
 f(x) = x - 2 - \frac{4}{x+1} & f'(x) = 1 - \frac{-4}{(x+1)^2} = 1 + \frac{4}{(x+1)^2} \\
 g(x) = x^4 + x^3 + 7x^2 & \text{la dérivée est toujours positive saut en -1 où elle n'existe pas.} \\
 h(x) = 2x^4 - 8x^2 + 1 & g'(x) = 4x^3 + 3x^2 + 14x = x(4x^2 + 3x + 14) \\
 i(x) = \frac{2x-5}{x-3} & \Delta = 9 - 224 = -215 \text{ La parenthèse est donc toujours positive.} \\
 j(x) = 2x + 2 + \frac{3}{2x+1} & h'(x) = 8x^3 - 16x = 8x(x^2 - 2) = 8x(x - \sqrt{2})(x + \sqrt{2}) \text{ au passage h est paire} \\
 & i'(x) = \frac{2(x-3)-(2x-5)1}{(x-3)^2} = \frac{2x-6-2x+5}{(x-3)^2} = \frac{-1}{(x-3)^2} \\
 & j'(x) = 2 + \frac{-3 \cdot 2x}{(2x+1)^2} = \frac{2(2x+1)^2 - 6}{(2x+1)^2} = \frac{2((2x+1)^2 - \sqrt{3}^2)}{(2x+1)^2} = \frac{2(2x+1-\sqrt{3})(2x+1+\sqrt{3})}{(2x+1)^2}
 \end{array}$$



x	$-\infty$	$\frac{1}{2}$	$\frac{2}{3}$	$+\infty$
-6	-	-	-	-
$x - \frac{1}{2}$	-	-	0	+
$x - \frac{2}{3}$	-	0	+	+
$d'(x)$	-	0	+	0
$d(x)$				

x	$-\infty$	0	$+\infty$
$4x^2 + 3x + 14$	+	+	
x	-	0	+
$g'(x)$	-	0	+
$g(x)$			

x	$-\infty$	-1	$+\infty$
$(x+1)^2$	+	0	+
$f'(x)$	+		+
$f(x)$			

x	$-\infty$	3	$+\infty$
$(x-3)^2$	+	0	+
$i'(x)$	-		-
$i(x)$			

x	$-\infty$	$-\sqrt{2}$	0	$\sqrt{2}$	$+\infty$
$8x$	-	-	0	+	+
$x - \sqrt{2}$	-	-	-	0	+
$x + \sqrt{2}$	-	0	+	+	+
$h'(x)$	-	0	+	0	-
$h(x)$					

x	$-\infty$	$\frac{-1 - \sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{-1 + \sqrt{3}}{2}$	$+\infty$
$(2x+1)^2$	+	+	0	+	+
$2x + 1 - \sqrt{3}$	-	-	-	0	+
$2x + 1 + \sqrt{3}$	-	0	+	+	+
$j'(x)$	+	0	-		-
$j(x)$					