

Correction du contrôle n°1 (complexes)

Barème : Ex 1 (4pts) Ex 2 (8pts + 2pts) Ex 3 (2pts) Ex 4 (6pts)

Exercice 1

Mettre sous la forme algébrique les complexes suivants :

$$z_1 = 12e^{-i\frac{\pi}{6}} \times \frac{1}{4}e^{i\frac{\pi}{3}}$$

$$= 12 \frac{1}{4} e^{-i\frac{\pi}{6}} \times e^{i\frac{\pi}{3}}$$

$$= 3e^{+i\frac{\pi}{6}} = 3(\cos\frac{\pi}{6} + i \sin\frac{\pi}{6})$$

$$= 3(\frac{\sqrt{3}}{2} + i\frac{1}{2}) = \frac{3\sqrt{3}}{2} + i\frac{3}{2}$$

$$z_2 = 12e^{-i\frac{\pi}{6}} + 6e^{i\frac{\pi}{6}}$$

$$= 12(\cos\left(-\frac{\pi}{6}\right) + i \sin\left(-\frac{\pi}{6}\right)) + 6(\cos\frac{\pi}{6} + i \sin\frac{\pi}{6})$$

$$= 12(\frac{\sqrt{3}}{2} - i\frac{1}{2}) + 6(\frac{\sqrt{3}}{2} + i\frac{1}{2})$$

$$= 6\sqrt{3} - 6 + 3\sqrt{3} + i3 = 9\sqrt{3} - i3$$

Exercice 2

Soit $z_1 = 5 + 5i$ et $z_2 = 3 - i\sqrt{3}$,

$$1) |z_1| = \sqrt{5^2 + 5^2} = 5\sqrt{2} \quad \cos\theta = \frac{5}{5\sqrt{2}} = \frac{\sqrt{2}}{2} \text{ et } \sin\theta = \frac{5}{5\sqrt{2}} = \frac{\sqrt{2}}{2} \text{ donc } \theta = \frac{\pi}{4} \quad z_1 = [5\sqrt{2}; \frac{\pi}{4}]$$

$$|z_2| = \sqrt{3^2 + \sqrt{3}^2} = 2\sqrt{3} \quad \cos\theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \text{ et } \sin\theta = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{-1}{2} \text{ donc } \theta = \frac{\pi}{6} \quad z_2 = [2\sqrt{3}; \frac{-\pi}{6}]$$

$$2) z_1^8 = [5\sqrt{2}; \frac{\pi}{4}]^8 = [(5\sqrt{2})^8; 8\frac{\pi}{4}] = [6 250 000; 2\pi] = 6 250 000,$$

$$3) \frac{z_1}{z_2} = \frac{[5\sqrt{2}; \frac{\pi}{4}]}{[2\sqrt{3}; \frac{-\pi}{6}]} = \left[\frac{5\sqrt{2}}{2\sqrt{3}}; \frac{\pi}{4} - \frac{-\pi}{6} \right] = \left[\frac{5\sqrt{6}}{6}; \frac{5\pi}{12} \right]$$

$$\text{bonus)} \quad z_1 \times z_2 = \left[5\sqrt{2}; \frac{\pi}{4} \right] \left[2\sqrt{3}; \frac{-\pi}{6} \right] = \left[5\sqrt{2} \times 2\sqrt{3}; \frac{\pi}{4} + \frac{-\pi}{6} \right] = \left[10\sqrt{6}; \frac{\pi}{12} \right]$$

$$z_1 \times z_2 = (5 + 5i)(3 - i\sqrt{3}) = 5(3 + \sqrt{3}) + i5(3 - \sqrt{3})$$

$$\text{Ainsi } \cos\left(\frac{\pi}{12}\right) = \frac{5(3+\sqrt{3})}{10\sqrt{6}} = \frac{(3+\sqrt{3})\sqrt{6}}{2\sqrt{6}\sqrt{6}} = \frac{3\sqrt{6}+\sqrt{18}}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\sin\left(\frac{\pi}{12}\right) = \frac{5(3-\sqrt{3})}{10\sqrt{6}} = \frac{(3-\sqrt{3})\sqrt{6}}{2\sqrt{6}\sqrt{6}} = \frac{3\sqrt{6}-\sqrt{18}}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

Exercice 3

Utiliser les formules d'Euler pour transformer en sommes les expressions suivantes :

$$f(\theta) = \sin(3\theta).\cos(2\theta) = \left(\frac{e^{i3\theta} - e^{-i3\theta}}{2i} \right) \left(\frac{e^{i2\theta} + e^{-i2\theta}}{2} \right) = \frac{e^{i5\theta} + e^{i\theta} - e^{-i\theta} - e^{-i5\theta}}{4i} \\ = \frac{1}{2} \left(\frac{e^{i5\theta} - e^{-i5\theta}}{2i} + \frac{e^{i\theta} - e^{-i\theta}}{2i} \right) = \frac{1}{2} (\sin(5\theta) + \cos(2\theta))$$

$$h(\theta) = \cos^5 \theta = \left(\frac{1}{2} (1 + \cos(2\theta)) \right)^2 \cos(\theta) = \frac{1}{4} (1 + 2\cos(2\theta) + \cos^2(2\theta)) \cos(\theta)$$

$$= \frac{1}{4} \left(1 + 2\cos(2\theta) + \frac{1}{2} (1 + \cos(4\theta)) \right) \cos(\theta)$$

$$= \frac{1}{8} (3 + 4\cos(2\theta) + \cos(4\theta)) \cos(\theta) = \frac{1}{8} \left(3 + 4 \frac{e^{i2\theta} + e^{-i2\theta}}{2} + \frac{e^{i4\theta} + e^{-i4\theta}}{2} \right) \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$= \frac{1}{8} \left(\frac{8 + 8e^{i2\theta} + 8e^{-i2\theta} + e^{i4\theta} + e^{-i4\theta}}{2} \right) = \frac{1}{16} \left(\frac{8e^{i\theta} + 4e^{i3\theta} + 4e^{-i\theta} + e^{i5\theta} + e^{-i3\theta} + 8e^{-i\theta} + 4e^{i\theta} + 4e^{-i3\theta} + e^{i3\theta} + e^{-i5\theta}}{2} \right)$$

$$= \frac{1}{16} (12\cos(2\theta) + 5\cos(3\theta) + \cos(5\theta))$$

Exercice 4

Résoudre les équations suivantes dans \mathbb{C}

$$1) z^2 + 2z + 10 = 0 \quad \Delta = 2^2 - 4 \times 10 = -36 \quad z_1 = \frac{-2-i6}{2} = -1 - 3i \quad z_2 = \frac{-2+i6}{2} = -1 + 3i$$

$$2) z^2 + 9z - 7 = 0 \quad \Delta = 9^2 + 4 \times 7 = 109 \quad z_1 = \frac{-9-\sqrt{109}}{2} \quad z_2 = \frac{-9+\sqrt{109}}{2}$$

$$3) (z-3)(5-2i) = (3+z)i \quad 5z - 2iz - 15 + 6i = 3i + zi \quad 5z - 2iz - zi = 15 - 6i - 3i$$

$$z(5-3i) = (15-9i) \quad z = \frac{15-9i}{5-3i} = \frac{(15-9i)(5+3i)}{(5-3i)(5+3i)} = \frac{75+45i-45i+27}{25+9} = \frac{102}{34} = \frac{61}{17} = 3$$

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